

Assignment 3: Logical Agents

Due 2/22 at 11:59pm.

Please work individually and use the [submission instructions](#).

Q1. Basics (2 points)

State if each of the sentences below are unsatisfiable, satisfiable, or valid.

- $P \Rightarrow Q$
- $P \Rightarrow \neg Q$
- $P \Rightarrow P$
- $P \Leftrightarrow \neg P$
- $P \Rightarrow (Q \Rightarrow P)$

Q2. Manipulating propositional sentences (3 points)

Show that any CNF formula (conjunction of clauses, where each clause is a disjunct of literals) can be converted to a 3-CNF formula (conjunction of clauses where each clause is at most a disjunction of 3 literals) where the 3-CNF formula is true in the same models that the original CNF sentence was true. **Extra Credit (1 point)**. Show that a CNF formula can be converted to a 3-CNF formula where each clause has exactly 3 disjuncts.

Q3. Propositional Resolution (2 points)

- Consider the sentence *Heads, I win. Tails, you lose*. Design a propositional KB that represents the sentence (create the propositions and rules required). Then use propositional resolution to prove that I always win.

Q4. Propositional Puzzle (3 points)

Write out the facts as sentences in Propositional Logic, and use propositional resolution to solve the crime.

1. There are three suspects for a murder: Adams, Brown, and Clark.
2. Adams says “I didn’t do it. The victim was old acquaintance of Brown’s. But Clark hated him.”
3. Brown states “I didn’t do it. I didn’t know the guy. Besides I was out of town all the week.”
4. Clark says “I didn’t do it. I saw both Adams and Brown downtown with the victim that day; one of them must have done it.”
5. Assume that the two innocent men are telling the truth, but that the guilty man might not be.

Q5. First Order Logic (Translation for FOL to Natural Language) (2 points)

Translate the following FOL sentences into colloquial English. You may assume the obvious meanings of all constants.

1. $\forall x \text{ Hesitates}(x) \Rightarrow \text{Lost}(x)$
2. $\neg \exists x \text{ Business}(x) \wedge \text{Like}(x, \text{Showbusiness})$

3. $\neg \forall x \text{ Glitters}(x) \Rightarrow \text{Gold}(x)$
4. $\exists x \text{ Mushroom}(x) \wedge (\forall z \neg (z = x) \Rightarrow \neg \text{Mushroom}(z))$

Q6. First Order Logic (Translation from Natural Language) (3 points)

Translate the following sentences. Use the following vocabulary.

- **Male (x)** means that the object denoted by x is male.
 - **Female (x)** means that the object denoted by x is female.
 - **Vegetarian (x)** means that x is vegetarian.
 - **Butcher (x)** means that x is a butcher.
 - **Like (x, y)** means that x likes y.
1. No man is both a butcher and a vegetarian.
 2. All men except butchers like vegetarians.
 3. The only vegetarian butchers are women.
 4. No man likes a woman who is vegetarian.
 5. No woman likes a man who does not like all vegetarians.