1 Bayes’ Nets (12 points)

Consider the following pairs of Bayes’ nets. If the two networks have identical conditional independences, write same, along with writing one of their shared independences (or none if they assert none). If the two networks have different conditional independences, write different, along with writing an independence that one has but not the other. For example, in the following case you would answer as shown:

\[
\text{different, right has } A \perp\!\!\!\!\!\perp B \mid \emptyset.
\]

(a) (1/2 pt)

(b) (1/2 pt)

(c) (1 pt)

(d) (1 pt)
The next parts involve computing various quantities in the network below. These questions are designed so that they can be answered with a minimum of computation. If you find yourself doing a copious amount of computation for each part, step back and consider whether there is simpler way to deduce the answer.

(e) (1/2 pt) $P(+a, -b, +c, -d)$

(f) (1/2 pt) $P(+b)$

(g) (1 pt) $P(+a | +b)$

(h) (1 pt) $P(+d | +a)$

(i) (1 pt) $P(+d | +a, +c)$
Next, consider the factors created by the variable elimination algorithm.

(j) (1 pt) Assuming no evidence, what factor is created by joining on the variable $B$?

(k) (1 pt) Again with no evidence, what is the set of factors remaining after joining on $B$ and then eliminating $B$? You only need to write their types (e.g. $P(A|B)$) and not their values.

(l) (1 pt) Assume you know that $D = +d$ and want to compute $P(C|d)$. What factors will remain after all eliminations have occurred, but before the final joining and normalizing has occurred?

Consider computing the following quantities in this network using various methods:

(i) $P(A|+b, +c, +d)$  
(ii) $P(C|d)$  
(iii) $P(D|a)$  
(iv) $P(D)$

(m) (1 pt) Which query is least expensive using inference by enumeration? Justify your answer.

(n) (1 pt) Will the advantage of likelihood weighting over rejection sampling be greater for (ii) or (iii) (in terms of number of samples required)? Briefly justify.
2 VPI: Catch the Cheater (9 points)

You work for a casino where the most popular game involves flipping a coin and seeing if it lands heads up. You are in charge of catching cheaters, who use two sided coins, which always come up heads, instead of standard fair coins. Assume that the prior probability of a gambler being a cheater is 1/16 and that a cheater always uses an unfair coin.

(a) (1 pt) Draw a Bayes’ net model over the variables C (cheater) and F₁ to Fₙ (N consecutive coin flips) in which the flips are conditionally independent given the coin type.

(b) (1 pt) In this model, what is the probability that a gambler is a cheater if you observe 4 flips, all heads?

As a gambler leaves the casino, you can either accuse them of having cheated or let them pass. Imagine that catching a cheater is worth +5 points, falsely accusing a non-cheater is worth -10, passing on a non-cheater is worth 0, and passing on a cheater is worth -1. For the next problems, you may leave your answers in fractional form.

(c) (1 pt) What is the optimal action if the gambler’s record for the night was 4 heads, and what is the expected utility of that action?

(d) (2 pt) In this model, what is the probability that a fifth flip will be heads again?
(e) (1 pt) What is the value of information of the outcome of a fifth flip? (Time saver: Given 5 heads, the MEU action is to stop the gambler, with EU = 10/47.)

(f) (2 pt) Assume you have evidence $E = e$ and that the utility $U(S, A)$ depends on a random variable $S$ and the action $A$. Prove that the value of information of observing new evidence $E'$ will be $\geq 0$.

(g) (1 pt) Prove that the value of information will be zero if the MEU action $a$ under $e$ is also the MEU action for all possible values of the augmented evidence $e, e'$. 
3 HMMs and Particle Filtering (9 points)

Your new apartment is haunted! A ghost is sometimes present causing rattling 75% of the time (rather than silence). Of course, even when the ghost is absent, there is still a 25% chance of rattling. Your research on ghostly migration suggests that every day there is a 10% chance that your ghost will leave if present or return if absent. When you moved in on day 1, you know that there was no ghost from the move-in ectoplasmic scan (you also heard silence). However, every day since, you have been hearing rattling.

(a) (1 pt) What is the probability that the ghost is present on day 2 given silence on day 1 and rattling on day 2?

(b) (2 pt) What is the probability that the ghost is present on day N, given silence on day 1 and rattling on days 2 through N?

(c) (1 pt) What is the probability that the ghost is present on day ∞, assuming there is rattling every day after day 1?
With all this rattling, you can’t concentrate hard enough to do exact inference! You decide to track the ghost using sampling.

(d) (1 pt) If you run rejection sampling (not particle filtering!) on the network up to day 4, what is the chance that a sample will be rejected?

(e) (1 pt) If you run likelihood weighting on the network up to day 4, what is the weight of a sample in which the state sequence is absent, present, present, present?

(f) (1 pt) If you run particle filtering and begin with 100 particles on absent and none on present on day 1, is it certain that those 100 particles will always all be on the same state for each future day. Justify.

New research suggests that ghosts always stay for exactly 13 days before leaving. Once gone, there is still a 10% chance of return on each day (though they are always gone for at least one day before returning).

(g) (2 pt) Describe how to change the HMM to reflect the assumptions of this new scenario.