1 Campus Layout (★)

You are asked to determine the layout of a new, small college. The campus will have three structures: an administration building (A), a bus stop (B), a classroom (C), and a dormitory (D). Each building must be placed somewhere on the grid below. The following constraints must be satisfied:

(i) The bus stop (B) must be adjacent to the road.
(ii) The administration building (A) and the classroom (C) must both be adjacent to the bus stop (B).
(iii) The classroom (C) must be adjacent to the dormitory (D).
(iv) The administration building (A) must not be adjacent to the dormitory (D).
(v) The administration building (A) must not be on a hill.
(vi) The dormitory (D) must be on a hill or adjacent to the road.
(vii) All buildings must be in different grid squares.

Here, “adjacent” means that the buildings must share a grid edge, not just a corner.

(a) Let the variables A, B, C, and D each range over the set of locations on the grid. Express the description above as unary and binary constraints over these variables. Implicit statements in precise but evocative notation such as different(X,Y) are acceptable.
(b) Cross out eliminated values to show the domains of all variables after unary constraints and arc consistency have been applied (but no variables have been assigned).

A \[ (1,1) (1,2) (1,3) (2,1) (2,2) (2,3) \]
B \[ (1,1) (1,2) (1,3) (2,1) (2,2) (2,3) \]
C \[ (1,1) (1,2) (1,3) (2,1) (2,2) (2,3) \]
D \[ (1,1) (1,2) (1,3) (2,1) (2,2) (2,3) \]

(c) Cross out eliminated values to show the domains of the variables after $B = (1, 3)$ has been assigned and arc consistency has been rerun.

A \[ (1,1) (1,2) (1,3) (2,1) (2,2) (2,3) \]
B \[ (1,1) (1,2) (1,3) (2,1) (2,2) (2,3) \]
C \[ (1,1) (1,2) (1,3) (2,1) (2,2) (2,3) \]
D \[ (1,1) (1,2) (1,3) (2,1) (2,2) (2,3) \]

(d) Give a solution for this CSP or state that none exist.
2 Trains (⋆)

A train scheduler must decide when trains $A$, $B$ and $C$ should depart. Once a train departs, it moves one space along its track each hour (in discrete jumps) until it arrives at its destination platform. Each train can depart at 1, 2 or 3 pm. The scheduler has two restrictions: All trains must leave at different times, and two trains should not both occupy crossing sections of track after any one hour time step is over. Note that train $A$ is two spaces long. Also note that the collision constraint is enforced only at the conclusion of every hour - time is discrete in this problem.

a) Describe the constraint satisfaction problem that, when solved, will tell the train scheduler when each train should depart. Let the variables $A$, $B$ and $C$ represent the departure times of the three trains.

b) Draw the constraint graph for the CSP you defined.

c) After selecting $A = 2$, cross out all values for $B$ and $C$ eliminated by forward checking.

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<tbody>
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d) Cross out all values eliminated by arc consistency before assigning any variables.

<table>
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<th>A</th>
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<td>1 2 3</td>
<td>1 2 3</td>
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e) After selecting $A = 2$, cross out all values for $B$ and $C$ eliminated by arc consistency.

<table>
<thead>
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<th>A</th>
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<td>2 1 2 3</td>
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f) Describe the execution of backtracking search using forward checking and the minimum remaining values (MRV) and least constraining values (LCV) heuristics. Specifically, in what order are the variables assigned and what values do they take? Start by assigning variable $A$. You may not need to fill all the lines below:

1) variable $A$ is assigned value .
2) variable is assigned value .
3) variable is assigned value .
4) variable is assigned value .
5) variable is assigned value .
6) variable is assigned value .

Note: Lines (2) and (3) may be switched.
3 Search as a CSP

Consider the following generic search problem formulation with finitely many states:

- **States**: there are $d + 2$ states: $\{s_s, s_g\} \cup \{s_1, \ldots, s_d\}$
- **Initial state**: $s_s$
- **Successor function**: $\text{Succ}(s)$ generates at most $b$ successors
- **Goal test**: $s_g$ is the only goal state
- **Step cost**: each step has a cost of 1

(a) Suppose an optimal solution exists with cost $n$. What is the tightest upper bound on $n$ in terms of the quantities defined above?

(b) Suppose we must solve this search problem using BFS, but with limited memory. Specifically, assume we can only store $k$ states during search. Give an upper bound on $n$ for which the search will fit in the available memory (do not worry about off-by-one errors here, but give the tightest bound possible). Note that in the worst case, the distinction between tree and graph search is not important here.

(c) Would any other search procedure allow problems with substantially deeper solutions to be solved? Either argue why not, or give a method along with an improved bound on $n$. 

(d) If we knew the exact value of \( n \), we could formulate a CSP whose complete assignment specifies an optimal solution path \( (X_0, X_1, \ldots, X_n) \) for this search problem. Give binary and/or unary constraints which guarantee that a satisfying assignment is a valid solution to the original search problem.

**Variables:** \( X_0, X_1, \ldots, X_n \)  
**Domains:** \( \text{Dom}(X_i) = \{s_s, s_g\} \cup \{s_1, \ldots, s_d\} \quad \forall \ i \in \{0, 1, \ldots, n\} \)  
**Constraints:**

(e) Assume the branching factor \( b \) is much less than \( d \). How can the successor function be used to efficiently enforce the consistency of an arc \( X_i \rightarrow X_{i-1} \)? (Reminder: Enforcing the consistency of this arc prunes values from the domain of \( X_i \), not \( X_{i-1} \).)

(f) After reducing the domains of any variables with unary constraints, suppose we then make all arcs \( X_i \rightarrow X_{i-1} \) consistent, processed in order from \( i = 1 \) to \( n \). Next, we try to assign variables in reverse order, from \( X_n \) to \( X_0 \), using backtracking DFS. Why is this a particularly good variable ordering?