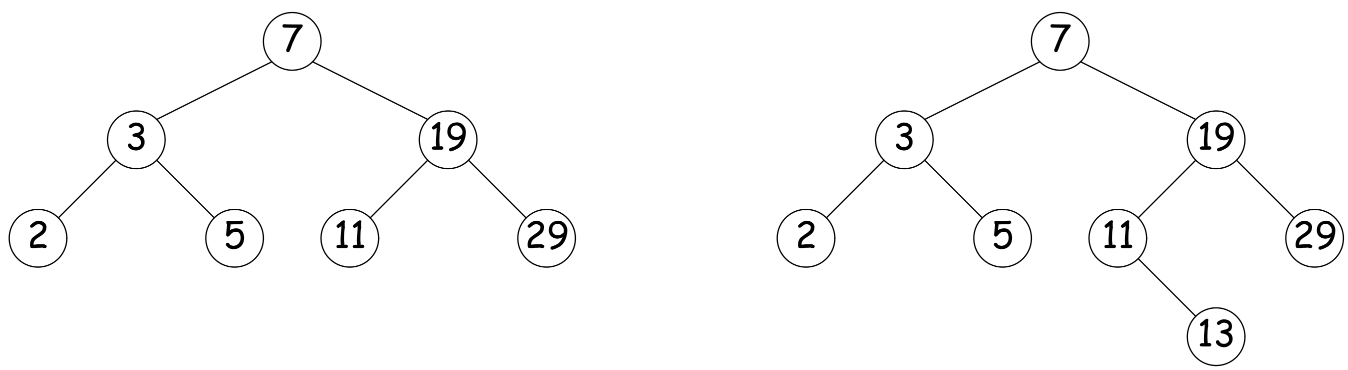
Lecture #22: Search Trees and Sets, Part II

Adding (Adjoining) a Value

- Must add values to a search tree in the right place: the place tree_find would try to find them.
- For example, if we add 17 to the search tree on left, we get the one on the right:



- Simplest always to add at the bottom (leaves) of the tree.

Non-destructive Add

- Broadly, there are two styles for dealing with structures that change over time:
  - *Non-destructive* operations preserve the prior state of the structure and create a new one.
  - *Destructive* operations, as a side effect, *may* modify the previous structure, losing information about its previous contents.

```
def tree_add(T, x):
    """Assuming T is a binary search tree, a new binary search tree
    that contains all previous values in T, plus X
    (if not previously present)."""
    if T.is_empty:
        return __________
    elif x == T.label:
        return _
    elif x < T.label:
        return ______________________________________________
    else:
        return ______________________________________________
```

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    if T.is_empty:
        return BinTree(x)
    elif x == T.label:
        return T
    elif x < T.label:
        return BinTree(T.label, tree_add(T.left, x), T.right)
    else:
        return BinTree(T.label, T.left, tree_add(T.right, x))
```

Destructive Operations

- Destructive operations can be appropriate in circumstances where
  - We want speed: avoid the work of creating new structures.
  - The same data structure is referenced from multiple places, and we want all of them to be updated.
- First requires that we add capabilities to our class:

```
class BinTree(Tree):
    def set_left(self, newval):
        """Assuming NEWVAL is a BinTree, sets SELF.left to NEWVAL."""
        ...

    def set_right(self, newval):
        """Assuming NEWVAL is a BinTree, sets SELF.right to NEWVAL."""
        ...
```

Destructive Add

- Destructive add looks very much like the non-destructive variety.

```
def dtree_add(T, x):
    """Assuming T is a binary search tree, a binary search tree
    that contains all previous values in T, plus X
    (if not previously present). May destroy the initial contents
    of T."""
    if T.is_empty:
        return __________
    elif x == T.label:
        return _
    elif x < T.label:
        ________________________________
        return _
    else:
        __________________________________
        return _
```

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    that contains all previous values in T, plus X
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    if T.is_empty:
        return BinTree(x)
    elif x == T.label:
        return T
    elif x < T.label:
        T.set_left(dtree_add(T.left, x))
        return T
    else:
        T.set_right(dtree_add(T.right, x))
        return T
```

Binary Search Trees as Sets

- For data that has a well-behaved ordering relation (a *total ordering*), BinTree provides a possible implementation of Python's set type.
- x in S corresponds to tree_find(S, x)
- S.union({x}) or S + {x} correspond to tree_add(S, x)
- S.add(x) or S += {x} correspond to dtree_add(S, x)
- Actually, Python uses *hash tables* for its sets, which you'll see in CS61B (plug).

Problem: Make a Balanced Tree

- I have a sorted list, and would like to turn it into the best (shallowest) binary search tree that contains the same values.
- Hint: Getting a shallow tree requires making the two child subtrees of each node have equal numbers of values (± 1).

```
def list_to_tree(L):
    """Assuming L is a sorted list, a (nearly) balanced
    search tree containing exactly the values in L."""
    if ___________:
        return _______________
    else:
        root_index = ___________
        return
```

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def list_to_tree(L):
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    if len(L) == 0:
        return Tree.empty_tree
    else:
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    search tree containing exactly the values in L."""
    if len(L) == 0:
        return Tree.empty_tree
    else:
        root_index = len(L) // 2
        return BinTree(L[root_index],
                       list_to_tree(L[:root_index]),
                       list_to_tree(L[root_index+1:]))
```

Problem: Iterating Through All Values

- Iterating over a tree gives us only the children, at present.
- Could we get *all* the nodes or labels in a tree,
- ...and for binary search trees, could we get them in sorted order?
- All it takes is a method that returns an appropriate iterator or iterable, and we can write, e.g.,

```
for val in T.preorder_values():
    ...
```

- How would we do that?

```
class Tree:
    ...
    def preorder_values(self):
        return ?
```

- Here, ? could be a list of all values in the tree, which we've done already. What else?

Creating an Iterator (Review)

- As we've seen (Lecture 17), an *iterator* is an object that implements a method __next__ on itelf.
- When called, it should either return a value or raise StopException.
- An *iterable* is an object that either
  - Implements a method __iter__(self) that returns an iterator, or
  - Implements a method __getitem__(self, k) that returns item number k (or raises an exception).
- Many methods and constructs take iterables, including **for** clauses, map, reduce, zip, and many others.
- When given an iterable, these create a new iterator from it (using __iter__), which allows one pass over the data.

Iterating Over a Binary Tree: Strategy

- To create an iterator on a tree, consider this reimplementation of tree_to_list_preorder from Lecture 21 (for binary trees):

```
def tree_to_list_preorder(T):
    """The list of all labels in T, listing the labels
    of trees before those of their children, and listing their
    children left to right (preorder)."""
    if T.is_empty:
        return ()
    else:
        return (T.label,) + tree_to_list_preorder(T.left) \
                          + tree_to_list_preorder(T.right)
```

- Suppose that we wanted to return just the first item (T's label). What work would be left to do?
- Clearly, returning (iterating through) all the values in the left child and then on the right.
- To get the next value (after T's label), we'll need to *start* iterating through the left child, leaving *its* children to be processed.
- When the next tree in the queue is empty, discard it.

Iterating Over a Binary Tree: Data Structure

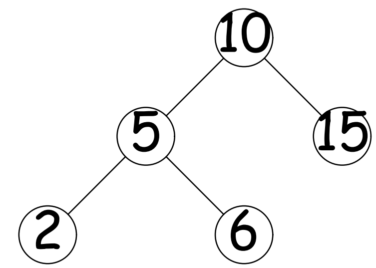
- So, to iterate over a tree, let's have our iterator consist of a *list of subtrees that still need iterating over*.

```
class BinTree(Tree):
    ...
    def __iter__(self): return tree_iter(self)
class tree_iter:
    def __init__(self, the_tree):
        self._work_queue = [ the_tree ]
    ...
    def __next__(self): ?

    # Standard hack: by making iterators implement __iter__, they
    # are themselves iterable, so you can use them in
    # for statements, etc.
    def __iter__(self): return self
```

Iterating Over a Binary Tree: Example

- Suppose that we create iter = T.__iter__() where T is



- Initially, iter._work_queue would contain just the tree rooted at the node labeled 10 (let's just say 'Tree 10' from now on).
- After the first call to iter.__next__(), which returns 10, iter._work_queue would contain [Tree 5, Tree 15]
- After the second call to iter.__next__(), which returns 5, iter._work_queue would contain [Tree 2, Tree 6, Tree 15]
- Then [Empty, Empty, Tree 6, Tree 15]