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CS61C : Machine Structures

Lecture 16 – Floating Point II

2004-10-06

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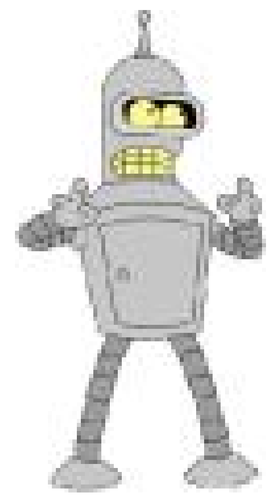
20 years from now...
1) We'll all have robot servants
or...
2) The world will be a smoking ruin



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Example: Representing 1/3 in MIPS

- 1/3

  $= 0.33333..._{10}$

  $= 0.25 + 0.0625 + 0.015625 + 0.00390625 + \ldots$

  $= 1/4 + 1/16 + 1/64 + 1/256 + \ldots$

  $= 2^{-2} + 2^{-4} + 2^{-6} + 2^{-8} + \ldots$

  $= 0.0101010101\ldots_2 * 2^0$

  $= 1.0101010101\ldots_2 * 2^{-2}$

  - Sign: 0
  - Exponent = -2 + 127 = 125 = 01111101
  - Significand = 0101010101…

0	0111 1101	0101 0101 0101 0101 0101 010

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Representation for ± ∞

- In FP, divide by 0 should produce ± ∞, not overflow.
- Why?
  - OK to do further computations with ∞ E.g., X/0 > Y may be a valid comparison
  - Ask math majors
- IEEE 754 represents ± ∞
  - Most positive exponent reserved for ∞
  - Significands all zeroes

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Special Numbers

- What have we defined so far? (Single Precision)

Exponent	Significand	Object
0	0	0
0	nonzero	???
1-254	anything	+/- fl. pt. #
255	0	+/- ∞
255	nonzero	???

- Professor Kahan had clever ideas; "Waste not, want not"
  - Exp=0,255 & Sig!=0 …

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Representation for Not a Number

- What is `sqrt(-4.0)`or `0/0`?
  - If ∞ not an error, these shouldn't be either.
  - Called <u>N</u>ot <u>a</u> <u>N</u>umber (NaN)
  - Exponent = 255, Significand nonzero
- Why is this useful?
  - Hope NaNs help with debugging?
  - They contaminate: op(NaN, X) = NaN

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Representation for Denorms (1/2)

- **Problem: There's a gap among representable FP numbers around 0**
  - **Smallest representable pos num:**

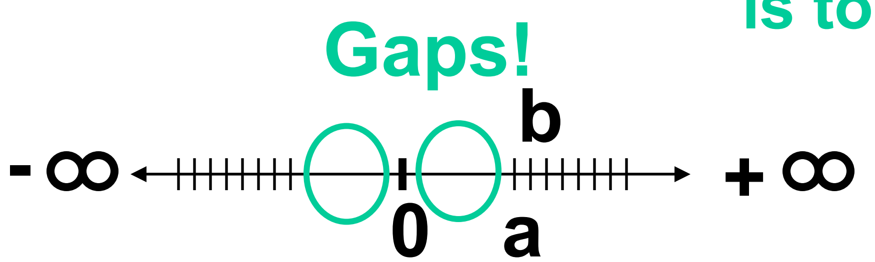
    $a = 1.0\ldots_2 * 2^{-126} = 2^{-126}$
  - **Second smallest representable pos num:**

    $b = 1.000\ldots\ldots1_2 * 2^{-126} = 2^{-126} + 2^{-149}$

  $a - 0 = 2^{-126}$

  $b - a = 2^{-149}$

Normalization and implicit 1 is to blame!



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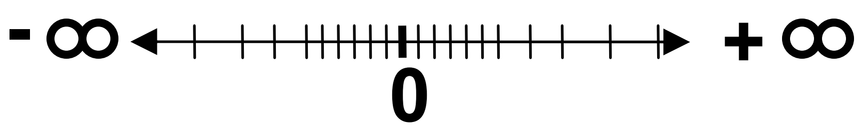
Representation for Denorms (2/2)

- **Solution:**
  - **We still haven't used Exponent = 0, Significand nonzero**
  - **Denormalized number: no leading 1, implicit exponent = -126.**
  - **Smallest representable pos num:**

    $a = 2^{-149}$
  - **Second smallest representable pos num:**

    $b = 2^{-148}$



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Overview

- Reserve exponents, significands:

Exponent	Significand	Object
0	0	0
0	nonzero	Denorm
1-254	anything	+/- fl. pt. #
255	0	+/- ∞
255	nonzero	NaN

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Rounding

- Math on real numbers $\Rightarrow$ we worry about rounding to fit result in the significant field.
- FP hardware carries 2 extra bits of precision, and rounds for proper value
- Rounding occurs when converting…
  - double to single precision
  - floating point # to an integer

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IEEE Four Rounding Modes

- Round towards + ∞
  - ALWAYS round "up": 2.1 ⇒ 3, -2.1 ⇒ -2
- Round towards - ∞
  - ALWAYS round "down": 1.9 ⇒ 1, -1.9 ⇒ -2
- Truncate
  - Just drop the last bits (round towards 0)
- Round to (nearest) even (default)
  - Normal rounding, almost: 2.5 ⇒ 2, 3.5 ⇒ 4
  - Like you learned in grade school
  - Insures fairness on calculation
  - Half the time we round up, other half down

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Integer Multiplication (1/3)

- **Paper and pencil example (unsigned):**

```
Multiplicand   1000      8
 Multiplier    x1001         9
                   1000
                  0000
                 0000
               +1000
               01001000
```

  - **m bits x n bits = m + n bit product**

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Integer Multiplication (2/3)

- In MIPS, we multiply registers, so:
  - 32-bit value x 32-bit value = 64-bit value
- Syntax of Multiplication (signed):
  - `mult` register1, register2
  - Multiplies 32-bit values in those registers & puts 64-bit product in special result regs:
    - puts product upper half in hi, lower half in lo
  - hi and lo are 2 registers separate from the 32 general purpose registers
  - Use `mfhi` register & `mflo` register to move from hi, lo to another register

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Integer Multiplication (3/3)

- **Example:**
  - **in C:** `a = b * c;`
  - **in MIPS:**
    - let b be $s2; let c be $s3; and let a be $s0 and $s1 (since it may be up to 64 bits)

```
mult $s2,$s3   # b*c
mfhi $s0       # upper half
of                     # product
into $s0
mflo $s1       # lower half of
               # product into $s1
```

- **Note: Often, we only care about the lower half of the product.**

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Integer Division (1/2)

- **Paper and pencil example (unsigned):**

```
                    1001    Quotient
Divisor 1000|1001010    Dividend
           -1000
               10
               101
               1010
              -1000
                  10   Remainder
              (or Modulo result)
```

- **Dividend = Quotient x Divisor + Remainder**

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Integer Division (2/2)

- Syntax of Division (signed):
  - `div` register1, register2
  - Divides 32-bit register 1 by 32-bit register 2:
  - puts remainder of division in `hi`, quotient in `lo`
- Implements C division (`/`) and modulo (%)
- Example in C: `a = c / d;`
  `b = c % d;`
- in MIPS: `a↔$s0;b↔$s1;c↔$s2;d↔$s3`

```
div  $s2,$s3     # lo=c/d, hi=c%d
mflo $s0         # get quotient
mfhi $s1         # get remainder
```


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Unsigned Instructions & Overflow

- MIPS also has versions of `mult`, `div` for unsigned operands:

```
multu
divu
```

  - Determines whether or not the product and quotient are changed if the operands are signed or unsigned.

- MIPS <u>does not</u> check overflow on ANY signed/unsigned multiply, divide instr
  - Up to the software to check `hi`

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FP Addition & Subtraction

- Much more difficult than with integers (can't just add significands)
- How do we do it?
  - De-normalize to match larger exponent
  - Add significands to get resulting one
  - Normalize (& check for under/overflow)
  - Round if needed (may need to renormalize)
- If signs ≠, do a subtract. (Subtract similar)
  - If signs ≠ for add (or = for sub), what's ans sign?
- Question: How do we integrate this into the integer arithmetic unit? [Answer: We don't!]

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MIPS Floating Point Architecture (

- Separate floating point instructions:
  - Single Precision: `add.s, sub.s, mul.s, div.s`
  - Double Precision: `add.d, sub.d, mul.d, div.d`
- These are far more complicated than their integer counterparts
  - Can take much longer to execute

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MIPS Floating Point Architecture (

- Problems:
  - Inefficient to have different instructions take vastly differing amounts of time.
  - Generally, a <u>particular piece of data will not change FP ⇔ `int`</u> within a program.
    - Only 1 type of instruction will be used on it.
  - Some programs <u>do no FP calculations</u>
  - It takes lots of hardware relative to integers to do FP fast

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MIPS Floating Point Architecture (

- 1990 Solution: Make a completely separate chip that handles only FP.
- Coprocessor 1: FP chip
  - contains 32 32-bit registers: `$f0`, `$f1`, …
  - most of the registers specified in `.s` and `.d` instruction refer to this set
  - separate load and store: `lwc1` and `swc1` ("load word coprocessor 1", "store …")
  - Double Precision: by convention, even/odd pair contain one DP FP number: `$f0`/`$f1`, `$f2`/`$f3`, … , `$f30`/`$f31`
    - Even register is the name

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MIPS Floating Point Architecture (

- 1990 Computer actually contains multiple separate chips:
  - Processor: handles all the normal stuff
  - Coprocessor 1: handles FP and only FP;
  - more coprocessors?… Yes, later
  - Today, FP coprocessor integrated with CPU, or cheap chips may leave out FP HW
- Instructions to move data between main processor and coprocessors:
  - `mfc0`, `mtc0`, `mfc1`, `mtc1`, etc.
- Appendix contains many more FP ops

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Peer Instruction 1

- Let X = # of floats between 1 and 2
- Let Y = # of floats between 2 and 3

```
1: X > Y
2: X < Y
3: X = Y
```


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Peer Instruction 2

1. Converting `float` -> `int` -> `float` produces same `float` number
2. Converting `int` -> `float` -> `int` produces same `int` number
3. FP add is associative: `(x+y)+z = x+(y+z)`

	ABC
1:	FFF
2:	FFT
3:	FTF
4:	FTT
5:	TFF
6:	TFT
7:	TTF
8:	TTT

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"And in conclusion…"

- Reserve exponents, significands:

Exponent	Significand	Object
0	0	0
0	nonzero	Denorm
1-254	anything	+/- fl. pt. #
255	0	+/- ∞
255	nonzero	NaN

- Integer `mult`, `div` uses `hi`, `lo` regs
  - `mfhi` and `mflo` copies out.
- Four rounding modes (to even default)
- MIPS FL ops complicated, expensive

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