inst.eecs.berkeley.edu/~cs61c

CS61C : Machine Structures

Lecture 23 – Combinational Logic Blocks

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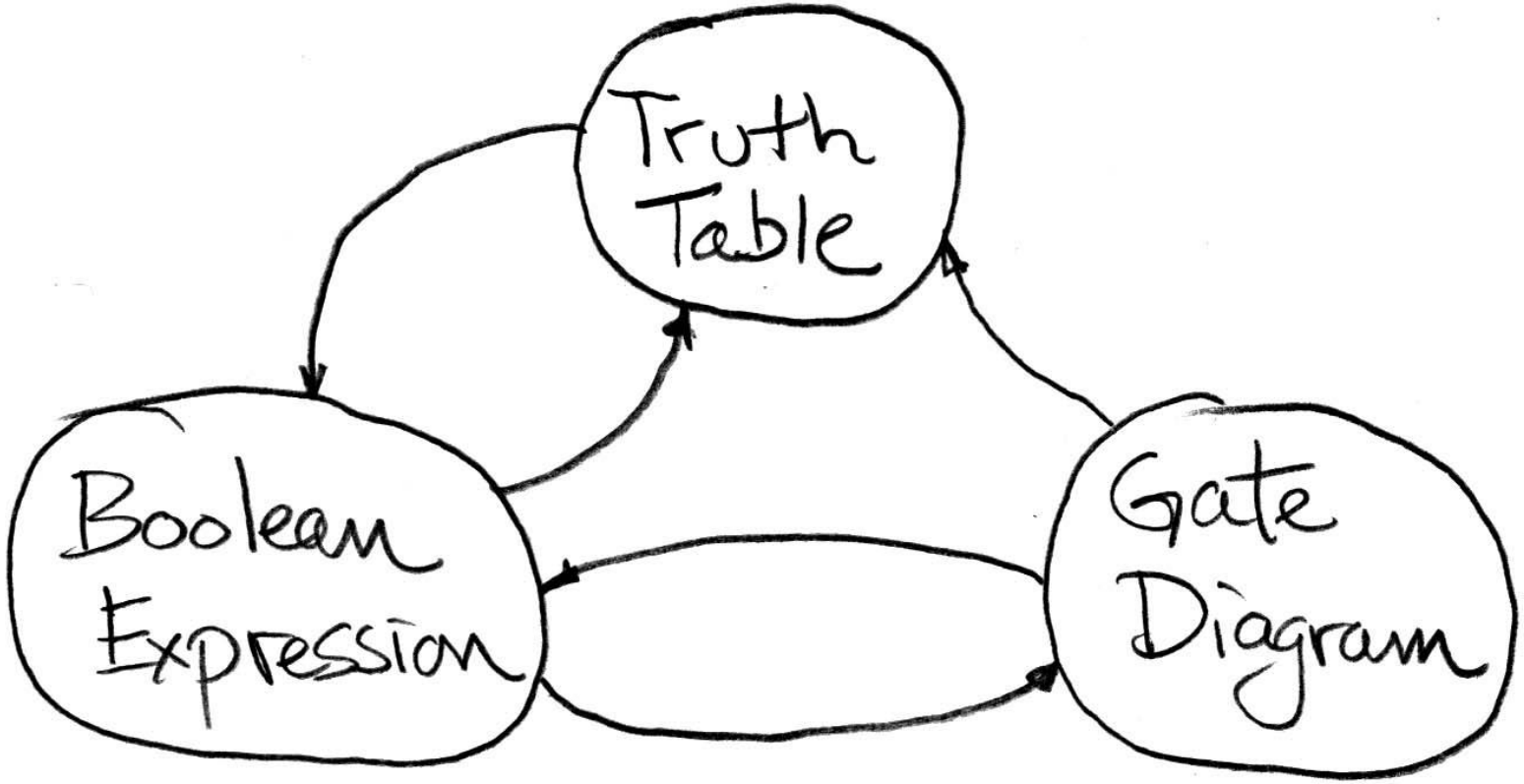
!!! ⇒ CeBIT 2005 in Hanover: A 7 MiPixel cell phone, and a 102” (2.6m) plasma TV!

www.cnn.com/2005/TECH/ptech/03/11/cebit.gadgets.reut

Review

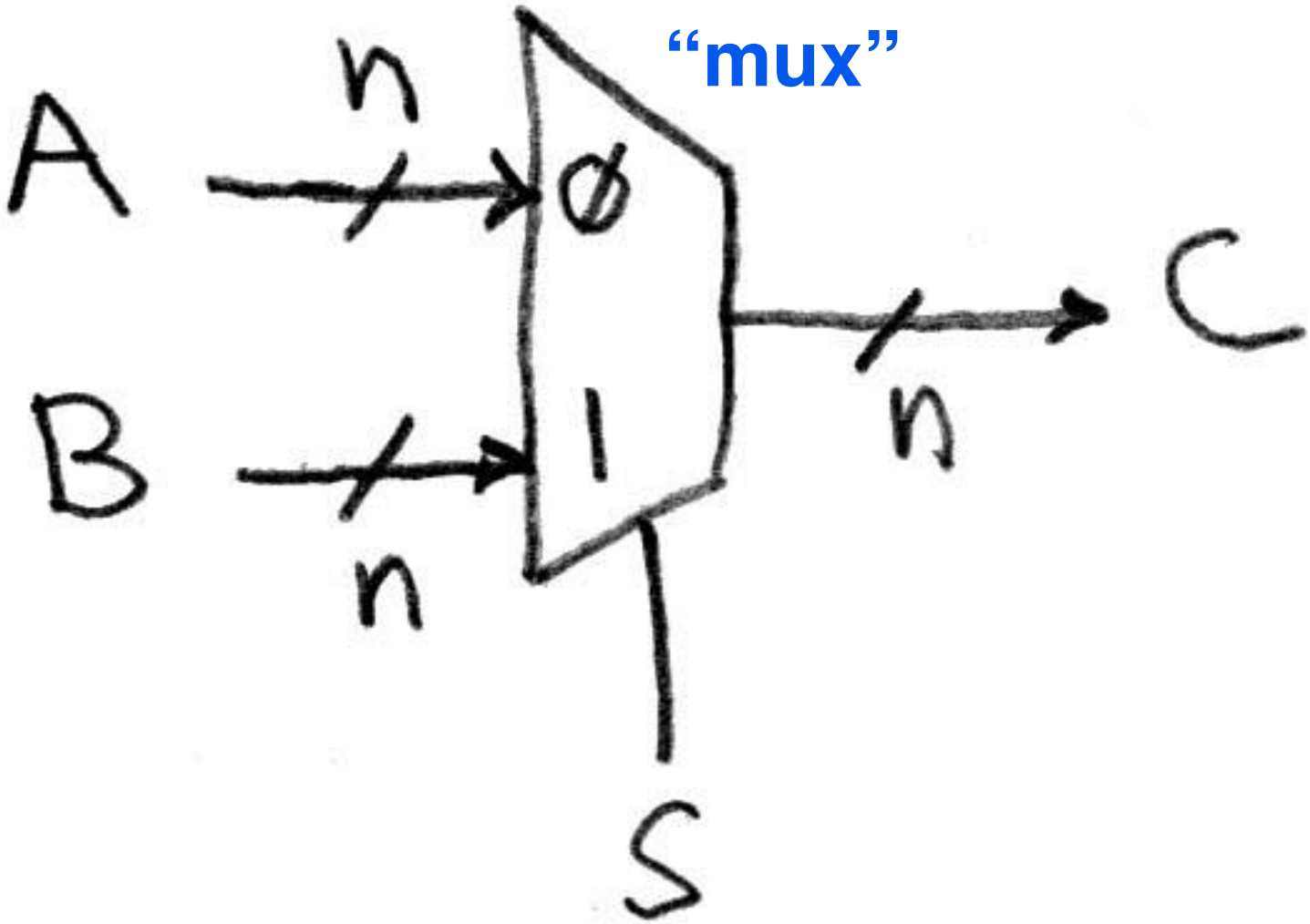
- **Use this table and techniques we learned to transform from 1 to another**



Today

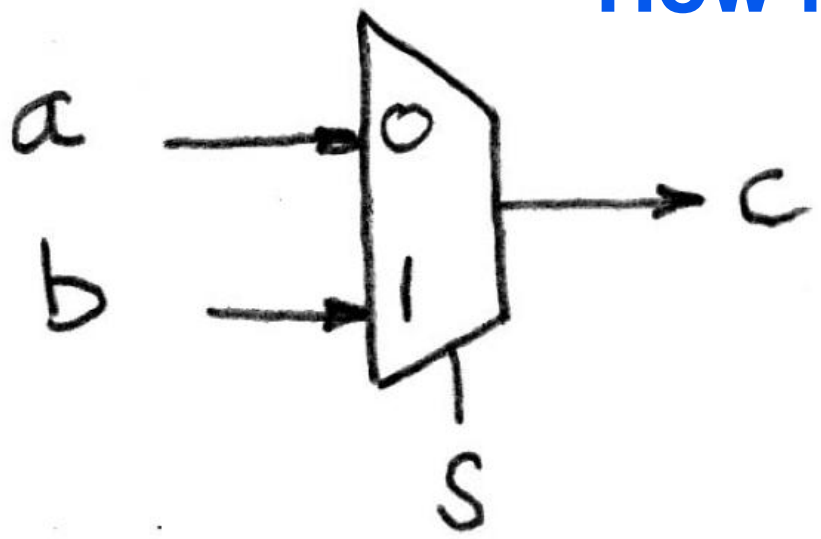
- Data Multiplexors
- Arithmetic and Logic Unit
- Adder/Subtractor

Data Multiplexor (here 2-to-1, n-bit-wide)



N instances of 1-bit-wide mux

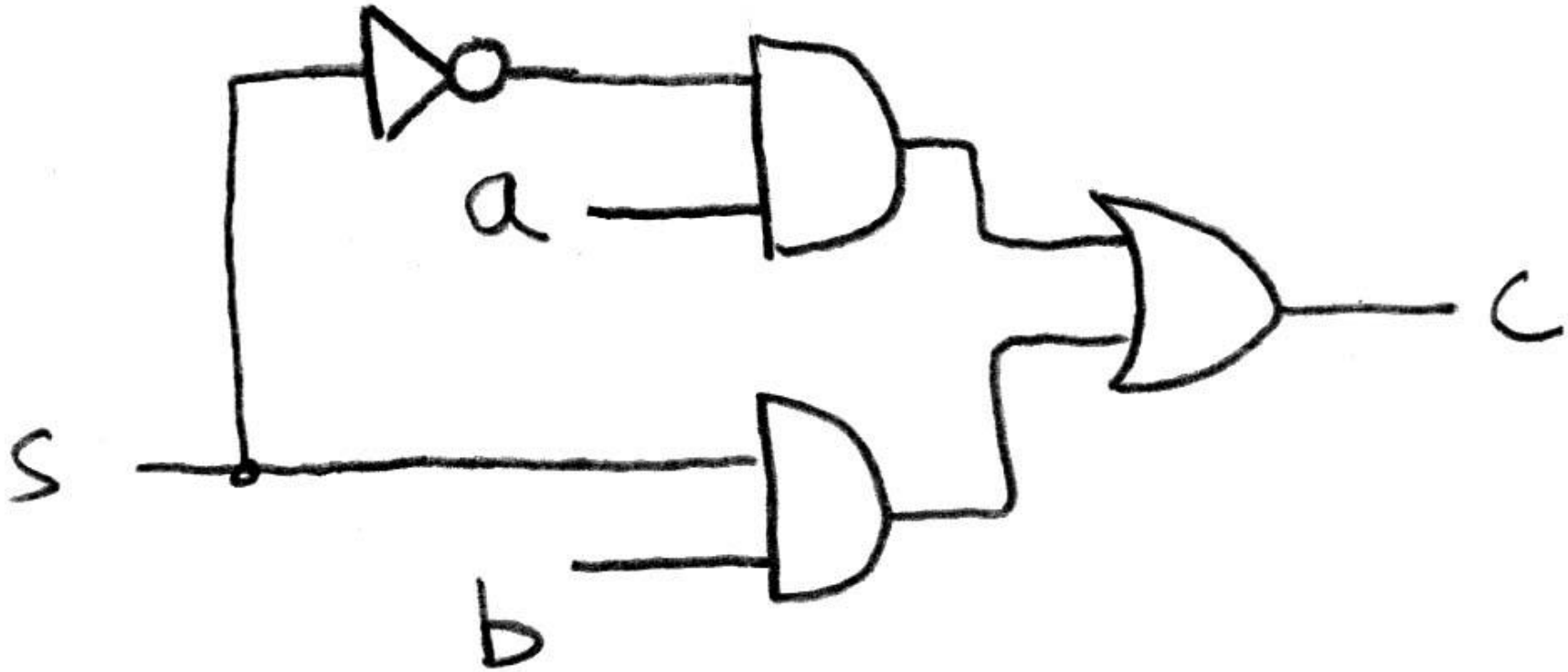
How many rows in TT?



$$
\begin{aligned}
c &= \overline{s}a\overline{b} + \overline{s}ab + s\overline{a}b + sab \\
&= \overline{s}(a\overline{b} + ab) + s(\overline{a}b + ab) \\
&= \overline{s}(a(\overline{b} + b)) + s((\overline{a} + a)b) \\
&= \overline{s}(a(1) + s((1)b) \\
&= \overline{s}a + sb
\end{aligned}
$$

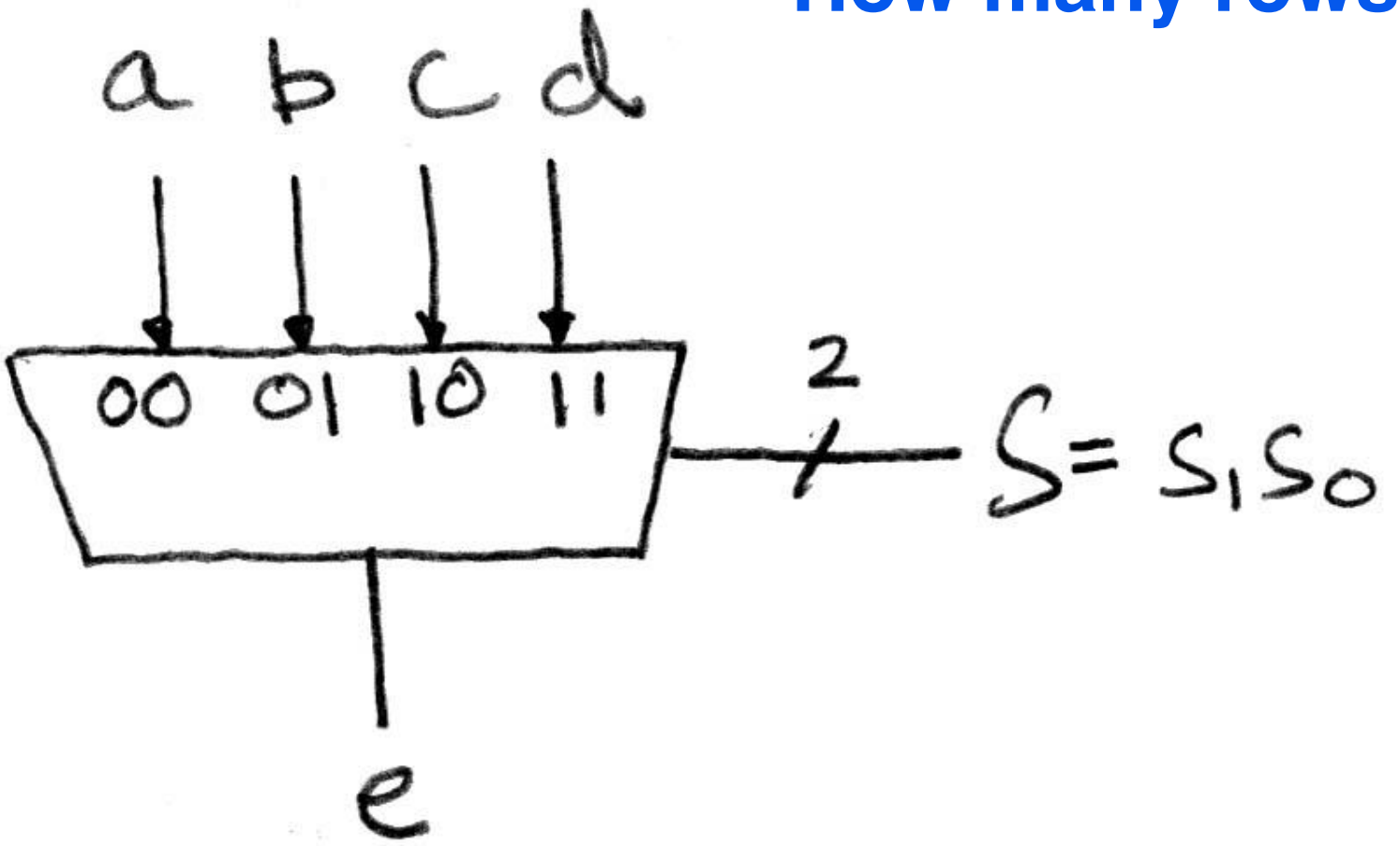
How do we build a 1-bit-wide mux?

$$\overline{s}a + sb$$



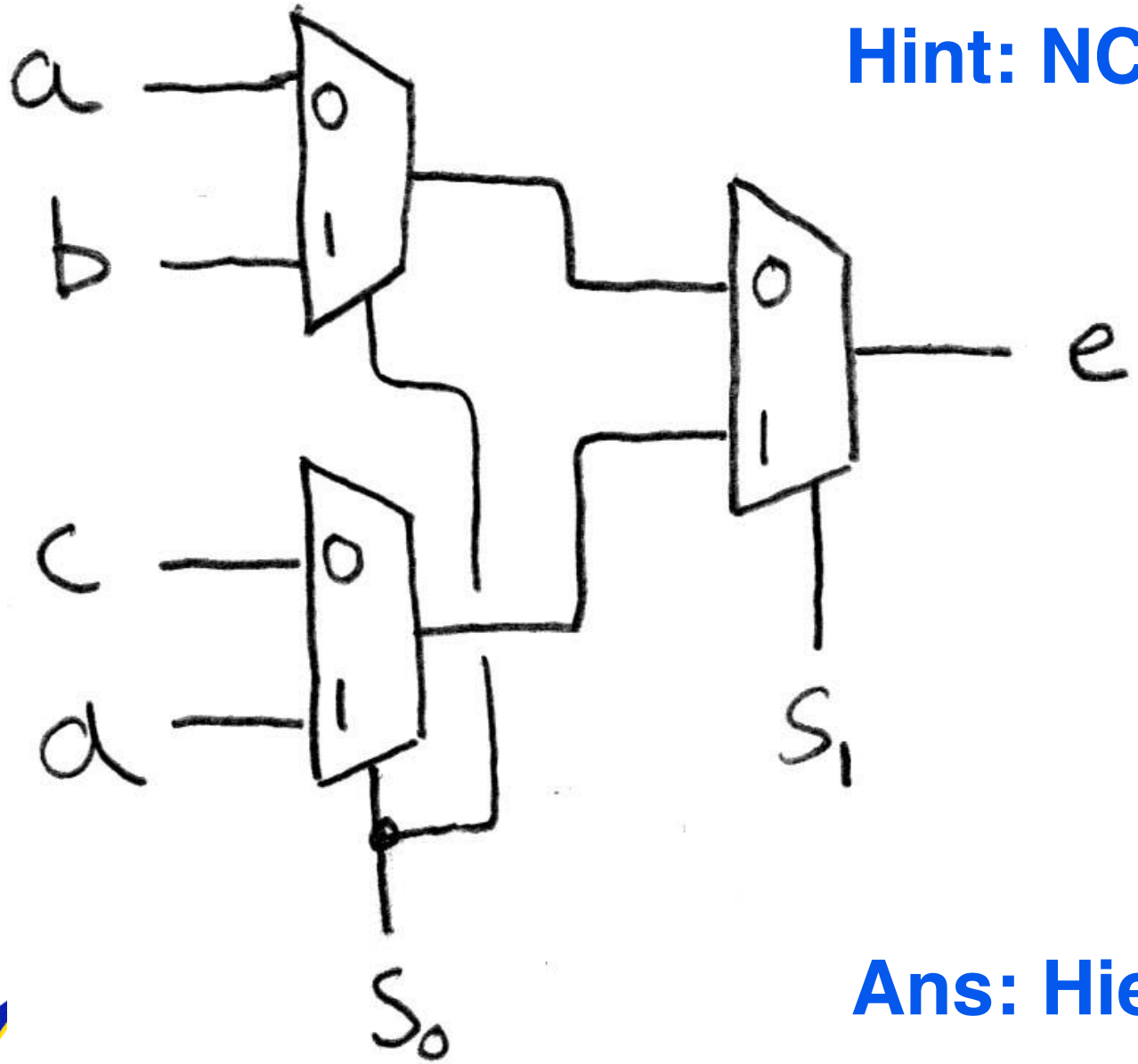
4-to-1 Multiplexor?

How many rows in TT?



$$e = \overline{s_1 s_0} a + \overline{s_1} s_0 b + s_1 \overline{s_0} c + s_1 s_0 d$$

Is there any other way to do it?



Hint: NCAA tourney!

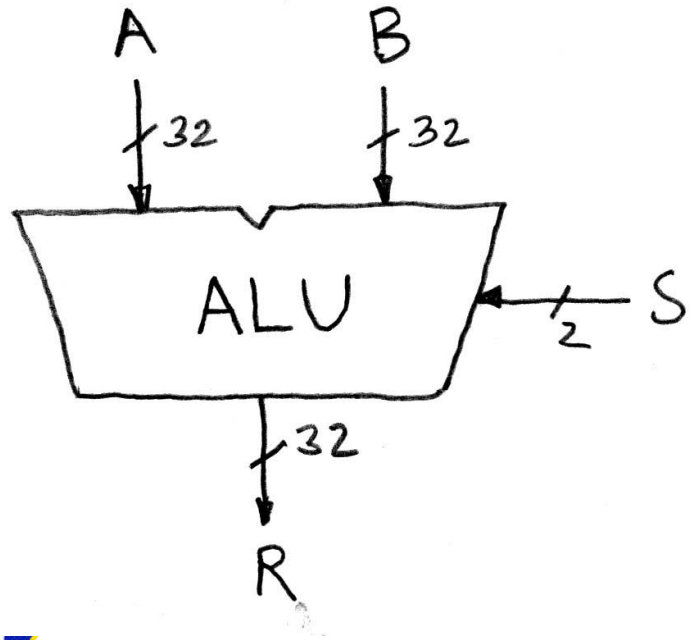
Ans: Hierarchically!

Administrivia

- Dan's Thursday OH cancelled (dentist)

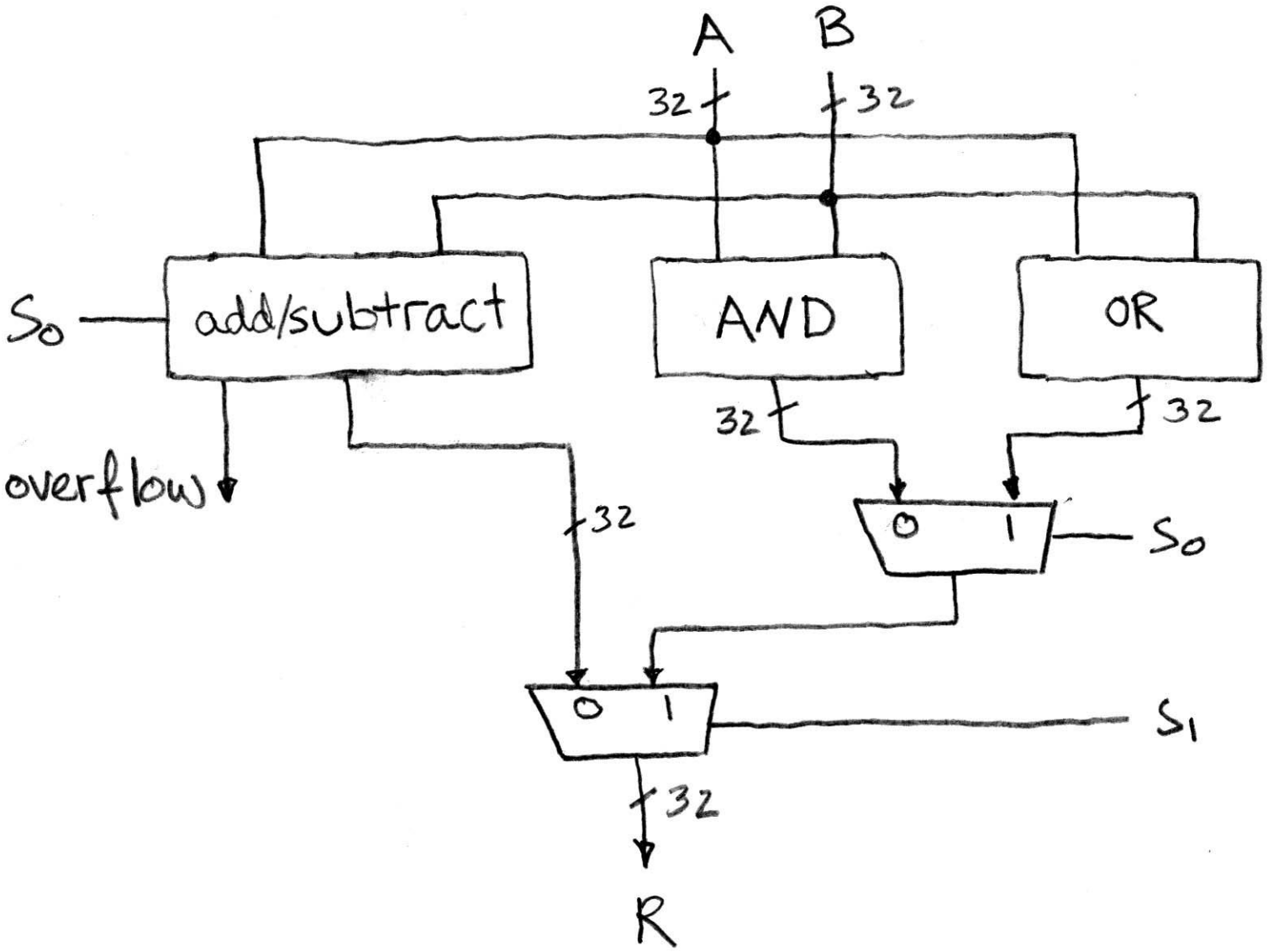
Arithmetic and Logic Unit

- **Most processors contain a special logic block called "Arithmetic and Logic Unit" (ALU)**
- **We'll show you an easy one that does ADD, SUB, bitwise AND, bitwise OR**



when S=00, R=A+B
when S=01, R=A-B
when S=10, R=A AND B
when S=11, R=A OR B

Our simple ALU



Adder/Subtracter Design -- how?

- Truth-table, then determine canonical form, then minimize and implement as we've seen before
- Look at breaking the problem down into smaller pieces that we can cascade or hierarchically layer

Adder/Subtracter – One-bit adder LSB…

$$
\begin{array}{cccc|c}
 & a_3 & a_2 & a_1 & a_0 \\
+ & b_3 & b_2 & b_1 & b_0 \\
\hline
 & s_3 & s_2 & s_1 & s_0
\end{array}
$$

a_0	b_0	s_0	c_1
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$s_0 =$$

$$c_1 =$$

Adder/Subtracter – One-bit adder (1/2)…

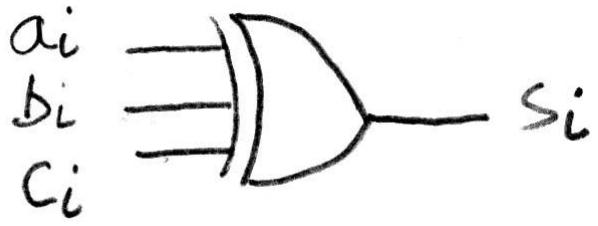
$$
\begin{array}{cccc}
 & a_3 & a_2 & \boxed{a_1} & a_0 \\
+ & b_3 & b_2 & \boxed{b_1} & b_0 \\
\hline
 & s_3 & s_2 & \boxed{s_1} & s_0
\end{array}
$$

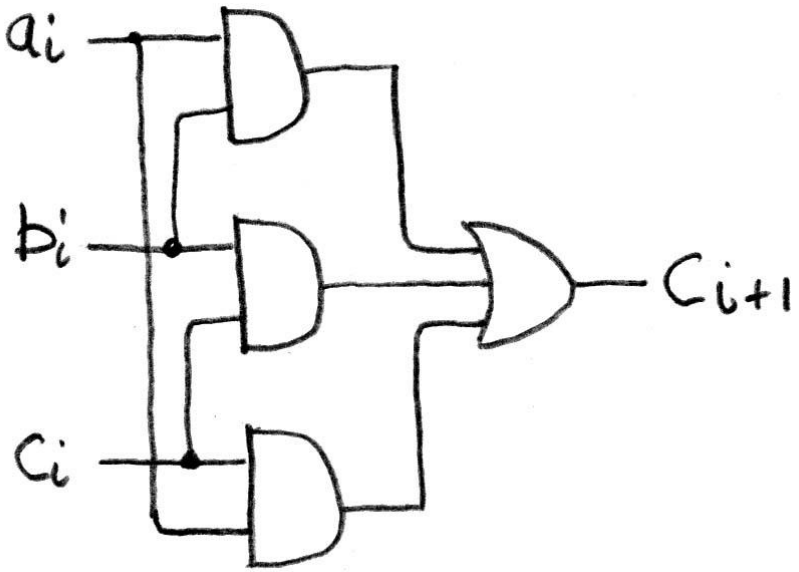
a_i	b_i	c_i	s_i	c_{i+1}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$s_i =$$

$$c_{i+1} =$$

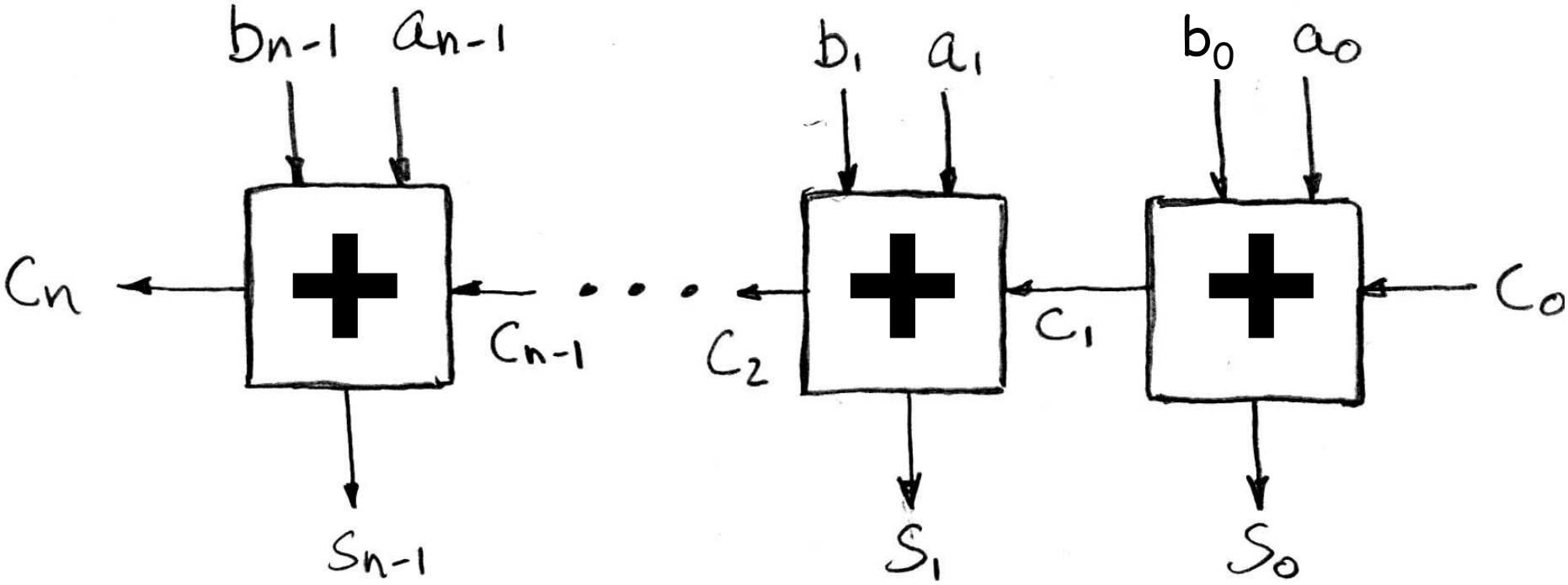
Adder/Subtracter – One-bit adder (2/2)…





$$s_i = \mathrm{XOR}(a_i, b_i, c_i)$$
$$c_{i+1} = \mathrm{MAJ}(a_i, b_i, c_i) = a_i b_i + a_i c_i + b_i c_i$$

N 1-bit adders ⇒ 1 N-bit adder

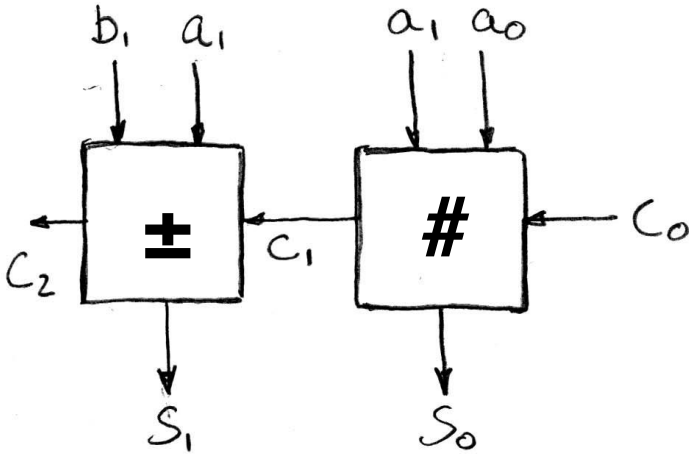


What about overflow?

Overflow = c_n?

What about overflow?

- Consider a 2-bit signed # & overflow:
  - 10 = -2 + -2 or -1
  - 11 = -1 + -2 only
  - 00 = 0 NOTHING!
  - 01 = 1 + 1 only



- Highest adder
  - C_1 = Carry-in = C_{in}, C_2 = Carry-out = C_{out}
  - No C_{out} or C_{in} ⇒ NO overflow!
  - C_{in}, and C_{out} ⇒ NO overflow!

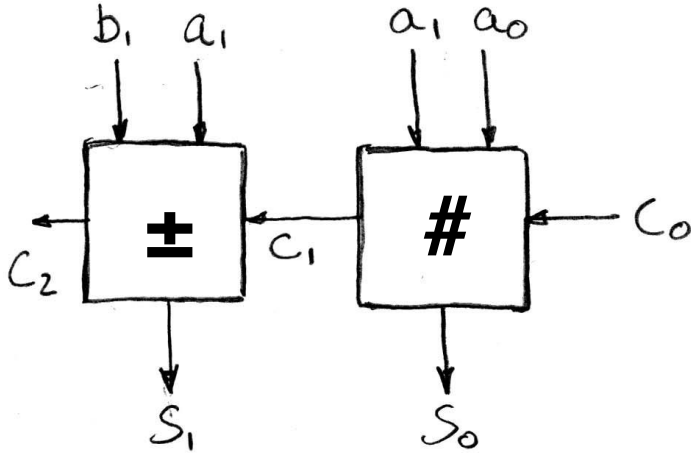
What op?

- C_{in}, but no C_{out} ⇒ A,B both > 0, overflow!
- C_{out}, but no C_{in} ⇒ A,B both < 0, overflow!

What about overflow?

- Consider a 2-bit signed # & overflow:

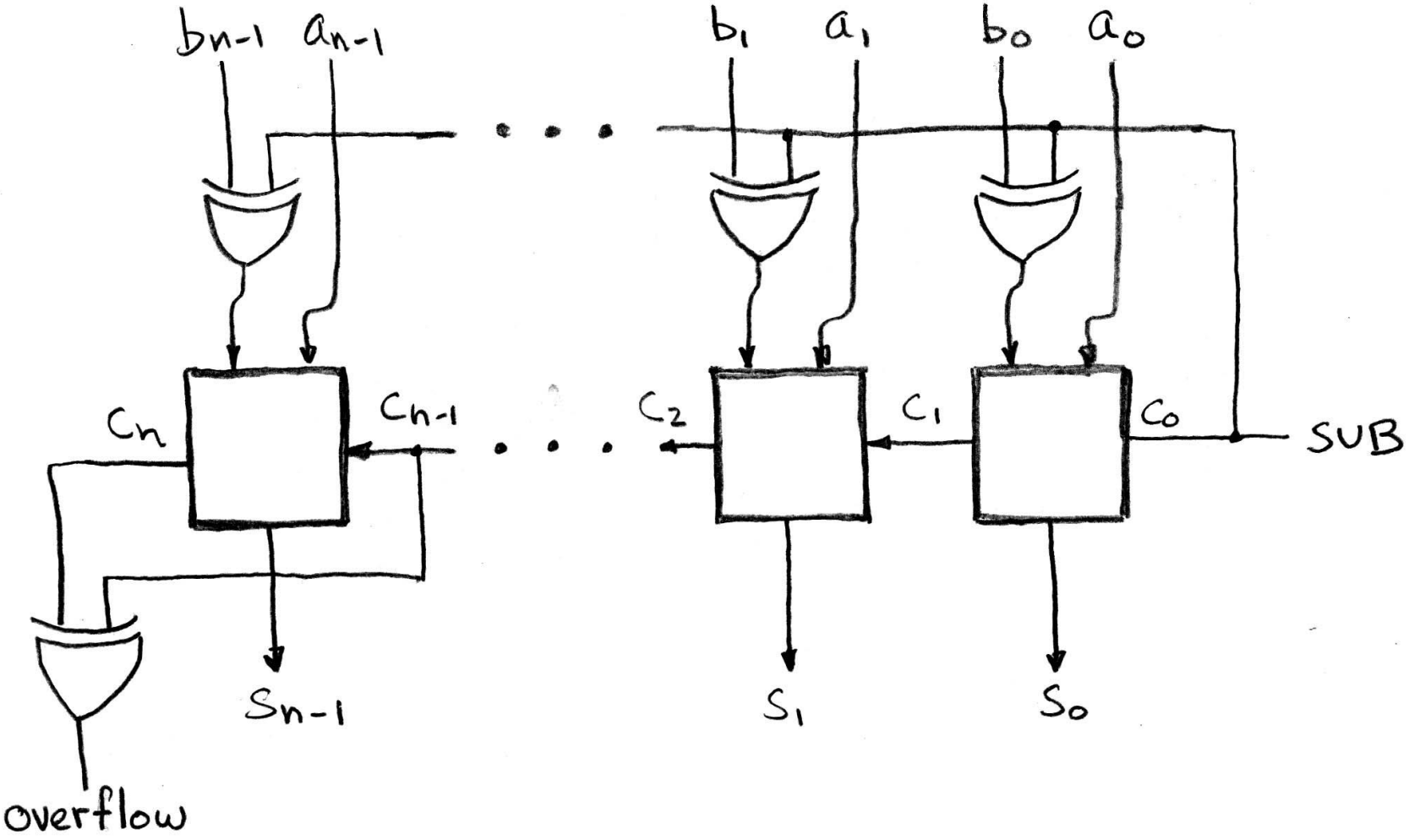
```
10 = -2
11 = -1
00 =  0
01 =  1
```



- Overflows when…
  - C_{in}, but no C_{out} ⇒ A,B both > 0, overflow!
  - C_{out}, but no C_{in} ⇒ A,B both < 0, overflow!

$$\text{overflow} = c_n \text{ XOR } c_{n-1}$$

Extremely Clever Subtractor



Peer Instruction

A. Truth table for mux with 4-bits of signals has 2^4 rows

B. We could cascade N 1-bit shifters to make 1 N-bit shifter for sll, srl

C. If 1-bit adder delay is T, the N-bit adder delay would also be T

	ABC
1:	FFF
2:	FFT
3:	FTF
4:	FTT
5:	TFF
6:	TFT
7:	TTF
8:	TTT

Peer Instruction Answer

"And In conclusion…"

- Use muxes to select among input
  - S input bits selects 2S inputs
  - Each input can be n-bits wide, indep of S
- Implement muxes hierarchically
- ALU can be implemented using a mux
  - Coupled with basic block elements
- N-bit adder-subtractor done using N 1-bit adders with XOR gates on input
  - XOR serves as conditional inverter