CS 70 Discrete Mathematics and Probability Theory
Fall 2011 Rao HW 1

Due Friday, September 2, 5:00pm

You *must* write up the solution set entirely on your own. You must never look at any other students' solutions (not even a draft), nor share your own solutions (not even a draft).

Please put your answer to each problem on its own sheet of paper, and paper-clip (don't staple!) the sheets of paper together. Label each sheet of paper with your name, your discussion section number (101–108), and "CS70–Fall 2011". Turn in your homework and problem x into the box labeled "CS70 – Fall 2010, Problem x" whereon the 2nd floor of Soda Hall. Failure to follow these instructions will likely cause you to receive no credit at all.

1. (20 pts.) Getting Started

1. You get an automatic 8 points for following directions (e.g., labelling every sheet of paper with your name and section, turning them in to the right box).
2. What has four wheels and flies?

   The answer is found on the Piazzza forum for CS 70. Look for the message from Satish Rao titled "The answer to question 1.1," and write down the answer you find there.

   (Why are we having you do this? The Piazzza forum is the way to get clarifications on homeworks, ask for help with topics from lecture that are confusing, etc. We want you to be familiar with how to access the Piazzza forum.)
3. What is the name of your TA? What is the name of the other TAs?
4. Two CS 70 students Alice and Bob decide to work in a group. They collaborate to figure out how to solve every question on the homework. Then, they split up the questions: Alice writes down the answers they came up with for questions 1 and 2, Bob writes down the answers they came up with for questions 3 and 4, and then they swap these papers and use them to finish their writeups. Under the CS 70 collaboration policy, is this OK, assuming they worked together on figuring out how to solve each question?

2. (20 pts.) Implications

Which of the following implications is true?

1. If $3+4=5$ then $3^2+4^2=5^2$.
2. If $3+4=7$ then $3^2+4^2=5^2$.
3. If $3+4=5$ then $3^2+4^2=7^2$.
4. If $3+4=7$ then $3^2+4^2=7^2$.
5. If any of this semester's CS 70 students are award-winning violinists, then $1+1=2$.
6. If Los Angeles is the state capital of California, then the trillionth digit of π is 7.

In part 6, $\pi = 3.14159\ldots$ denotes the ratio of the circumference of a circle to its diameter.

3. (20 pts.) If you show up on time, you won't have to work this hard!

You show up late to CS 70 lecture and come in the middle of a complex derivation involving the propositions P, Q, and R. From what you can see on the board, you're able to deduce that the following three propositions are true: $P \implies \neg P$, $Q \implies R$, $P \vee Q \vee \neg R$. Unfortunately, it looks like the definition of the propositions P, Q, R has already been erased.

1. Do you have enough information to deduce the truth value of P? If yes, what is the truth value of P?
2. Do you have enough information to deduce the truth value of Q? If yes, what is the truth value of Q?
3. David asks the class whether $(\neg Q \wedge R) \vee (Q \wedge \neg R)$ is true. Do you have enough information to deduce the truth value of this proposition? If yes, what is its truth value?

4. (20 pts.) Practice with quantifiers

Which of the following propositions is true? ($\mathbb{N} = \{0, 1, 2, \ldots\}$ denotes the set of natural numbers.)

1. $(\forall x \in \mathbb{N})(x^2 < 9) \implies (\forall x \in \mathbb{N})(x^2 < 10)$.
2. $(\forall x \in \mathbb{N})(x^2 < 10) \implies (\forall x \in \mathbb{N})(x^2 < 9)$.
3. $(\forall x \in \mathbb{N})(x^2 < 9 \implies x^2 < 10)$.
4. $(\forall x \in \mathbb{N})(x^2 < 10 \implies x^2 < 9)$.
5. $(\forall x \in \mathbb{N})(\exists y \in \mathbb{N})(x^2 < y)$.
6. $(\exists y \in \mathbb{N})(\forall x \in \mathbb{N})(x^2 < y)$.
7. $(\forall x \in \mathbb{N})(\exists y \in \mathbb{N})(x^2 < y \implies x < y)$.
8. $(\exists y \in \mathbb{N})(\forall x \in \mathbb{N})(x^2 < y \implies x < y)$.
9. $(\forall x \in \mathbb{N})(\exists y \in \mathbb{N})(x < y \implies x^2 < y)$.
10. $(\exists y \in \mathbb{N})(\forall x \in \mathbb{N})(x < y \implies x^2 < y)$.

5. (20 pts.) Grade these answers

You be the grader. Students have submitted the following answers to several exam questions. Assign each student answer either an A (correct yes/no answer, valid justification), a D (correct yes/no answer, invalid justification), or an F (incorrect answer). As always, $\pi = 3.14159\ldots$

1. **Exam question:** Is the following proposition true? $2\pi < 100 \implies \pi < 50$. Explain your answer.
   Student answer: Yes. $2\pi = 6.283\ldots$, which is less than 100. Also $\pi = 3.1459\ldots$ is less than 50. Therefore the proposition is of the form True $\implies$ True, which is true.
2. **Exam question:** Is the following proposition true? $2\pi < 100 \implies \pi < 50$. Explain your answer.
   Student answer: Yes. If $2\pi < 100$, then dividing both sides by two, we see that $\pi < 50$.
3. **Exam question:** Is the following proposition true? $2\pi < 100 \implies \pi < 49$. Explain your answer.
   Student answer: No. If $2\pi < 100$, then dividing both sides by two, we see that $\pi < 50$, which does not imply $\pi < 49$.
4. **Exam question:** Is the following proposition true? $\pi^2 < 5 \implies \pi < 5$. Explain your answer.
   Student answer: No, it is false. $\pi^2 = 9.87\ldots$, which is not less than 5, so the premise is false. You can't start from a faulty premise.

6. (20 pts.) Liars and Truthtellers

You find yourself on a desert island inhabited by two types of people: the Liars and the Truthtellers. Liars always lie, and Truthtellers always tell the truth. In all other respects, the two types are indistinguishable.

1. You meet a very attractive local and ask him/her on a date. The local responds, "I will go on a date with you if and only if I am a Truthteller." Is this good news? Explain your answer with reference to logical notation.
2. You are trying to find your way to the lagoon and encounter a local inhabitant on the road. Which of the following questions could you ask him in order to reliably deduce whether you are on the correct path? In each case, explain your answer with reference to logical notation.
   (a) If I were to ask you "If this is the way to the lagoon, what would you say?"
   (b) If I were to ask you, "If this is the way to the lagoon and you say "yes", can I believe you?"
   (c) If I were to ask somebody of the other type than yours, "If this is the way to the lagoon, what would that person say?"
   (d) "Is at least one of the following true? You are a Liar and this is the way to the lagoon; or you are a Truthteller and this is not the way to the lagoon."