CS 70	Discrete Mathematics and Probability Theory	
Fall 2011	Rao	HW 7

Due Friday, October 14, 5:00pm

You *must* write up the solution set entirely on your own. You must never look at any other students' solutions (not even a draft), nor share your own solutions (not even a draft).

Please put your answer to each problem on its own sheet of paper, and paper-clip (don't staple!) the sheets of paper together. Label each sheet of paper with your name, your class login, your discussion section number (101–108), and "CS70–Fall 2011". Turn in your homework and problem x into the box labeled "CS70 – Fall 2011, Problem x" whereon the 2nd floor of Soda Hall. Failure to follow these instructions will likely cause you to receive no credit at all.

1. (35 pts.) Sample Space and Events

Consider the sample space Ω of all outcomes from flipping a coin 4 times.

1. List all the outcomes in Ω. How many are there?
2. Let A be the event that the first flip is a Heads. List all the outcomes in A. How many are there?
3. Let B be the event that the third flip is a Heads. List all the outcomes in B. How many are there?
4. Let C be the event that the first and third flip are Heads. List all outcomes in C. How many are there?
5. Let D be the event that the first or the third flip is Heads. List all outcomes in D. How many are there?
6. Are the events A and B disjoint? Express C in terms of A and B. Express D in terms of A and B.
7. Suppose now the coin is flipped $n \geq 3$ times instead of 4 flips. Compute $|\Omega|, |A|, |B|, |C|, |D|$.
8. Your gambling buddy found a website online where he could buy trick coins that are heads or tails on both sides. He puts three coins into a bag: one coin that is heads on both sides, one coin that is tails on both sides, and one that is heads on one side and tails on the other side. You shake the bag, draw out a coin at random, put it on the table without looking at it, then look at the side that is showing. Suppose you notice that the side that is showing is heads. What is the probability that the other side is heads? Show your work. (Hint: the answer is NOT 1/2).

2. (15 pts.) Monty Hall revisited

In this variant of the Monty Hall problem, after the contestant has chosen a door, Monty asks another contestant to open one of the other two doors. That contestant, who also has no idea where the prize is, opens one of those two remaining doors at random, and (assuming that's what happens) you both see that there is no prize there. Monty now asks you if you wish to switch or stick with your original choice. What is your best strategy? Why? What is the probability you win if you stick, given that the other contestant's door didn't contain the prize? What is the probability you win if you switch, given that the other contestant's door didn't contain the prize?

3. (20 pts.) Rolling dice (conditional probability)

We roll two fair 6-sided dice. Each one of the 36 possible outcomes is assumed to be equally likely.

(a) Find the probability that doubles were rolled.

(b) Given that the roll resulted in a sum of 4 or less, find the conditional probability that doubles were rolled.

(c) Find the probability that at least one die is a 6.

(d) Given that the two dice land on different numbers, find the conditional probability that at least one die is a 6.