Midterm 1 is early.

- Before drop date.
- Before most other midterms in CS.

CS70: Lecture 2. Outline.

1. Propositions.
2. Propositional Forms.
3. Implication Again.
4. Wason's Experiment
5. Truth Tables
6. Quantifiers

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational **Proposition**

2+2 $= 4$

2+2 $= 3$

826th digit of pi is 4

Johny Depp is a good actor

Every even number > 2 is sum of 2 primes

$x + x$

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Propositional Forms.
Conjunction: $P \wedge Q$
Disjunction: $P \vee Q$
Implication: $P \implies Q$ or $\neg P \vee Q$
"not $(2+2=4)$" – a new statement that is false...
$2+2=3$ and $2+2=4$ – a new statement that is false

Propositional Forms....

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Not necessarily.
$P \implies Q$ and $Q = T$ does not mean $P = T$

Implication and English.
$P \implies Q$

- If P, then Q.

Implication and English.
$P \implies Q$

- If P, then Q.
- Q if P.

Implication and English.
$P \implies Q$

- If P, then Q.
- Q if P.
- P only if Q.
- P is sufficient for Q.
- Q is necessary for P.

Wason's experiment:1
Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.

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Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
- On side of each card is the person's destination, on the other is mode of travel.
- Consider the experimental rule: "If a person travels to Chicago, he/she flies." Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flies. Which cards do you need to flip to test the rule?

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- Consider the experimental rule: "If a person travels to Chicago, he/she flies." Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flies. Which cards do you need to flip to test the rule?

Answer: Bob's and Charlie's

Wason's experiment:2

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
- The cards say what each person has for dessert, the other what they did after dinner.
- New rule: "If a child has ice cream for dessert, he/she has to do the dishes after dinner."
- Cards: Alice: fruit, Bob: watched TV, Charlie: ice cream, Donna: did dishes

Now what cards do you have to flip?

Wason's experiment:2

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
- The cards say what each person has for dessert, the other what they did after dinner.
- New rule: "If a child has ice cream for dessert, he/she has to do the dishes after dinner."
- Cards: Alice: fruit, Bob: watched TV, Charlie: ice cream, Donna: did dishes

Now what cards do you have to flip?

== Answer: still Bob's and Charlie's

Wason's experiment: comment.

- Called "Wason selection task."
- Only 25% of population gets first right, but 65-80% gets second, even though they are the same!
- But if you switch to a converse, i.e. "If a child does dishes after dinner, he/she had ice cream for dessert," almost everyone gets it wrong! They look for cheaters, even though the rule doesn't tell them to.
- All mathematically the same.
- Intuition for us is not so good for mathematics....

Propositional Forms: tables.

P						

Propositional Forms: tables.

P					
T					

Propositional Forms: tables.

P					
T					
T					

Propositional Forms: tables.

P					
T					
T					
F					

Propositional Forms: tables.

P					
T					
T					
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Propositional Forms: tables.

P	Q				
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  Note: logically equivalent. $P \implies Q \equiv \not Q \implies \not P$.
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- If $P \implies Q$ and $Q \implies P$ is P if and only if Q or $P \iff Q$.

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Propositions?

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- $P(n) =$ "$\sum_{i=1}^{n} i = n(n+1)/2$."
- $R(x) =$ "$x > 2$"
- $G(n) =$ "n is even and the sum of two primes"

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    Equivalent to "$(0 = 0) \vee (1 = 1) \vee (2 = 4) \vee \ldots$"
  - "there exists a number greater than 5"
    $$\exists x \in N . x > 5 \quad \text{dot notation}$$

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- "the square of a number is always non-negative"

$$\forall x \in N . x^2 >= 0$$

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. Restrict domain using implication.

Note that we may omit universe if clear from context.

Quantifiers..not commutative.

- In English: "there is a natural number that is the square of every natural number", i.e the square of every natural number is the same number!

$$\exists y \in N \ \ \forall x \in N . y = x^2$$

  (false)
- In English: "the square of every natural number is a natural number"... $\forall x \in N \ \ \exists y \in N . y = x^2$ (true)

Quantifiers....negation...

- $$\neg(\forall x \in S)(P(x))$$

  means that there exist an x in S where $P(x)$ does not hold.

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The end for today.