Midterm 1 is early.

- ► Before drop date.
- ▶ Before most other midterms in CS.

CS70: Lecture 2. Outline.

- 1. Propositions.
- 2. Propositional Forms.
- 3. Implication Again.
- 4. Wason's Experiment
- 5. Truth Tables
- 6. Quantifiers

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Propositions: Statements that are true or false.

\sqrt{2} is irrational

2+2 = 4

2+2 = 3

826th digit of pi is 4

Johny Depp is a good actor

Every even number > 2 is sum of 2 primes

x + x

4+5
```

Proposition Proposition

Proposition Proposition Proposition

Proposition Proposition Proposition Proposition

Proposition Proposition Proposition Proposition Not a Proposition

Proposition Proposition Proposition Proposition Not a Proposition Proposition Not a Proposition.

Proposition Proposition Proposition Proposition Not a Proposition Not a Proposition. Not a Proposition. Propositional Forms. Conjunction: $P \land Q$ Disjunction: $P \lor Q$ Implication: $P \implies Q$ or $\neg P \lor Q$ "not (2+2=4)" – a new statement that is false... 2+2=3 and 2+2=4 – a new statement that is false

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 P ∨ Q
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My name is Satish and I love math. P ∧ Q P = "My name is Satish." Q = "I love math."

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 $P \implies Q$ and Q = T does not mean P = T

Implication and English. $P \implies Q$

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Implication and English.

- $P \implies Q$
 - If P, then Q.
 - ▶ *Q* if *P*.
 - ► *P* only if *Q*.
 - \triangleright *P* is sufficient for *Q*.
 - Q is necessary for P.

Wason's experiment:1 Suppose we have four cards on a table:

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- Consider the experimental rule: "If a person travels to Chicago, he/she flies." Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flies. Which cards do you need to flip to test the rule?

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Answer: Bob's and Charlie's

Suppose we have four cards on a table:

- ▶ 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
- The cards say what each person has for dessert, the other what they did after dinner.
- New rule: "If a child has ice cream for dessert, he/she has to do the dishes after dinner."
- Cards: Alice: fruit, Bob: watched TV, Charlie: ice cream, Donna: did dishes

Now what cards do you have to flip?

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Now what cards do you have to flip?

== Answer: still Bob's and Charlie's

Wason's experiment: comment.

- Called "Wason selection task."
- Only 25% of population gets first right, but 65-80% gets second, even though they are the same!
- But if you switch to a converse, i.e. "If a child does dishes after dinner, he/she had ice cream for dessert," almost everyone gets it wrong! They look for cheaters, even though the rule doesn't tell them to.
- All mathematically the same.
- Intuition for us is not so good for mathematics....

Ρ			

Ρ			
Т			

Ρ			
Т			
Т			

Ρ			
Т			
Т			
F			

Ρ			
Т			
Т			
F			
F			

Ρ	Q		
Т			
Т			
F			
F			

Ρ	Q T		
Т	Т		
Т			
F			
F			

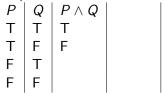
Ρ	Q T		
Т	Т		
Т	F		
F	Т		
F			

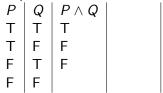
Ρ	Q T		
Т	Т		
Т	F		
F	Т		
F	F		

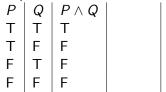
Ρ	Q T		
Т	Т		
Т	F		
F	Т		
F	F		



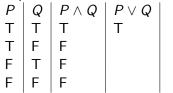




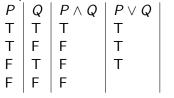


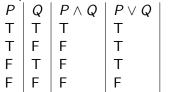


Ρ	Q	$P \wedge Q$ T	$P \lor Q$	
Т	Т	Т		
Т	F	F		
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- Converse of P ⇒ Q is Q ⇒ P.
 If fish die the plant pollutes.
 Not logically equivalent!
- If $P \implies Q$ and $Q \implies P$ is P if and only if Q or $P \iff Q$.

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$$P(n) = ::\sum_{i=1}^{n} i = n(n+1)/2.$$

•
$$R(x) = "x > 2"$$

• G(n) = "n is even and the sum of two primes"

Quantifiers.. there exists.

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► $(\exists x \in S)(P(x))$ means "P(x) is true for some x in S" $(\exists x \in N)(x = x^2)$ parenthesis notation Equivalent to " $(0 = 0) \lor (1 = 1) \lor (2 = 4) \lor ...$ " Quantifiers.. there exists.

(∃x ∈ S)(P(x)) means "P(x) is true for some x in S"
(∃x ∈ N)(x = x²) parenthesis notation
Equivalent to "(0 = 0) ∨ (1 = 1) ∨ (2 = 4) ∨ ..."
"there exists a number greater than 5"

 $\exists x \in N.x > 5$ dot notation

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"the square of a number is always non-negative"

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. Restrict domain using implication.

Note that we may omit universe if clear from context.

Quantifiers..not commutative.

In English: "there is a natural number that is the square of every natural number", i.e the square of every natural number is the same number!

$$\exists y \in N \ \forall x \in N.y = x^2$$

(false)

In English: "the square of every natural number is a natural number"... ∀x ∈ N ∃y ∈ N.y = x² (true)

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The end for today.