Section 1

1. Use truth tables to show that $\neg(A \vee B) \equiv \neg A \wedge \neg B$ and $\neg(A \wedge B) \equiv \neg A \vee \neg B$. These two equivalences are known as DeMorgan's Law.

2. Which of the following statements is/are true? (In the following, $Q(n)$ is the statement "n is divisible by 2." As usual, $\mathbb{N}$ denotes the set of natural numbers.)

(a) $\exists k \in \mathbb{N}, Q(k) \wedge Q(k+1)$.

(b) $\forall k \in \mathbb{N}, Q(k) \implies Q(k^2)$.

(c) $\exists x \in \mathbb{N}, \neg(\exists y \in \mathbb{N}, y < x)$.

3. Rewrite the following statements in propositional logic. (Use $\mathbb{N}$ to denote the set of natural numbers and $\mathbb{Z}$ to denote the set of integers. Also write $P(n)$ for the statement "n is odd".)

(a) For all natural numbers n, $2n$ is even.

(b) For all natural numbers n, n is odd if n^2 is odd.

(c) There are no integer solutions to the equation $x^2 - y^2 = 10$.

4. Which of the following implications is/are true?

(a) $\forall x \forall y\ P(x,y)$ implies $\forall y \forall x\ P(x,y)$.

(b) $\exists x \exists y\ P(x,y)$ implies $\exists y \exists x\ P(x,y)$.

(c) $\forall x \exists y\ P(x,y)$ implies $\exists y \forall x\ P(x,y)$.

(d) $\exists x \forall y\ P(x,y)$ implies $\forall y \exists x\ P(x,y)$.

Also, for the implication in part (c), what is its converse? And its contrapositive?

5. Complete the following expression so that it states that: "There is one and only one natural number n for which the proposition formula $P(n)$ holds."

$$(\exists n \in \mathbb{N}) \ldots$$