Section 7

1. (a) Suppose that instead of three doors, there are $n = 4$ doors in the Monty Hall puzzle. After you have chosen one door, the host, who knows what is behind each door, opens $k = 2$ losing doors and gives you the chance to change doors. What is the probability that you win by not changing? What is the probability that you win by changing the door?

(b) Repeat part (a) for a generic n and $k = n - 2$.

(c) Repeat part (a) for a generic n and $k = 1$.

2. (a) Suppose you record the birthdays of a large group of people, one at a time until you have found a match, i.e., a birthday that has already been recorded.

(i) What is the probability that it takes more than 20 people for this to occur?

(ii) What is the probability that it takes exactly 20 people for this to occur?

(b) Suppose you record the birthdays of a large group of people, one at a time, until you have found a person whose birthday matches your own birthday. What is the probability that it takes exactly 20 people for this to occur?

3. (a) Mr. Smith has two children, at least one of whom is a boy. What is the probability that the other is a girl?

(b) Mr. Smith has two children, at least one of whom is a boy born on a Tuesday. What is the probability that the other is a girl?

4. A doctor knows that a patient has exactly one of three diseases d_1, d_2, d_3. Before any test, she assumes an equal probability for each disease. She carries out a test that will be positive with probability 0.8 if the patient has d_1 , 0.6 if he has disease d_2, and 0.4 if he has disease d_3. Given that the outcome of the test was positive, what probabilities should the doctor now assign to the three possible diseases?

5. Decades ago Berkeley conducted a study on gender bias in graduate school admission. It was alleged that female applicants had a lower admission probability than male applicants, so they must be discriminated against. The gender bias study, on the other hand, claimed that in each[1] of the individual departments, female applicants were more likely to be admitted than male applicants. Could the allegation and the claim be both true?

Let's consider a simplified scenario where there are just two departments EE and CS. Suppose each applicant will apply to either EE or CS. Consider the following (made up) historical statistics, and evaluate the six conditional probabilities $\Pr[\text{accept} \mid \text{male}, \text{EE}], \Pr[\text{accept} \mid \text{female}, \text{EE}], \Pr[\text{accept} \mid \text{male}]$, etc. and determine whether the allegation and the claim are true.

EE	#apply	#accept
Men	2	0
Women	10	1

CS	#apply	#accept
Men	4	3
Women	4	4

[1] We have oversimplified slightly. For the true story, look up "Simpson's paradox" on Wikipedia.