Section 8

1. You toss a coin n times which has bias p for showing Heads.

(a) Give a probability space (Ω, P) to model this experiment.

(b) What is the probability that Heads comes up k times in the n tosses?

2. The disc containing the only copy of your homework got corrupted, and the disc got mixed up with three other corrupted discs that were lying around. So it is now equally likely that any of the four discs holds the corrupted remains of your homework. Your computer expert friend offers to have a look, and you know from past experience that his probability of finding your homework on any disc is 0.4 (assuming it is actually there). Given that he searches on disc 1 and cannot find your homework, what is the probability that your homework is on disc i, for $i = 1, 2, 3, 4$?

3. A roll of the dice
Consider a single roll of two dice, one red and one blue.

(a) Let R be the value of the red die. What is the distribution of R? What is the expectation of R?

(b) Let M be the larger of the numbers on the two dice. What is the distribution of M? What is the expectation of M?

(c) How do the distribution and expectation of M compare to that of R?

(d) Are the events $R = 6$ and $M = 6$ disjoint? Are they independent? What about the events $R = 6$ and $M = 1$?

[Hint: Recall that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ and $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.]

4. To pay or not to pay?
Reese Prosser never puts money in a 25-cent parking meter in Hanover. He assumes that there is a probability of 0.05 that he will be caught. Assume each offense that is caught costs him \$10. Under his assumptions:

(a) How does the expected cost of parking 10 times without paying the meter compare with the cost of paying the meter each time?

(b) If he parks at the meter 10 times, what is the probability that he will have to pay more than the total amount he could end up saving by not putting the money?

5. Monkey writing Shakespeare
A monkey types on a 26-letter keyboard, with all lowercase letters. Assume that the monkey chooses each character independently and uniformly at random.

(a) The monkey types a million six-letter words at random. What is the expected number of times the word "hamlet" is typed?
Let H_i be the event that the ith word typed is "hamlet." Are H_i and H_j independent for $i \neq j$?

(b) Now the monkey types a million characters at random. What is the expected number of times the sequence "hamlet" appears?
Letting H_i denote the event that the six-letter sequence that starts at the ith character is "hamlet," are H_i and H_j independent for $i \neq j$?

(c) Finally the monkey types a six-letter word at random. The monkey copies this word a million times to make a million-word text (meaning spaces between words are retained). What is the expected number of times the word "hamlet" appears in the text?
Letting H_i be the event that the ith word is "hamlet," are H_i and H_j independent for $i \neq j$?

(d) Let random variable X be the number of times "hamlet" appears. Think about what the distribution of X looks like in each of the three cases (a)-(c) above. Are any of the three distributions the same?