Section 10

1. A friend tells you about a course called "Laziness in Modern Society" that requires almost no work. You hope to take this course so that you can devote all of your time to CS70. At the first lecture, the professor announces that grades will depend only a midterm and a final. The midterm will consist of three questions, each worth 10 points, and the final will consist of four questions, also each worth 10 points. He will give an A to any student who gets at least 60 of the possible 70 points.

However, speaking with the professor in office hours you hear some very disturbing news. He tells you that to save time he will be grading as follows. For each student's midterm, he'll choose a real number randomly from a distribution with mean $\mu = 5$ and variance $\sigma^2 = 1$. He'll mark each of the three questions with that score. To grade the final, he'll again choose a random number from the same distribution, independent of the first number, and he'll mark all four questions with that score.

If you take the class, what will the mean and variance of your total class score be? Can you conclude that you have less than a 5% chance of getting an A? Why?

2. Imagine that you have n drawers in your filing cabinet, and that you left your term paper in drawer k with probability D_k . Furthermore, suppose that these drawers are so messy, that even if you correctly guess that the term paper is in drawer k, the probability that you find it is p_k. You can assume all the drawers take the same amount of time to search through. Also assume that each search through the same drawer is independent.

(a) What drawer should you start looking for your term paper in?

(b) Now, suppose you search for your paper in a particular drawer, say drawer i, and the search is unsuccessful. What drawer should you search for next? Can you think of a situation where the right thing is to search through the same drawer again?

(c) For each k, you have searched through drawer k a total of n_k times. Which drawer should you search through next?

3. For *any* random variables X and Y, linearity of expectation tells us that $\mathrm{E}[X+Y] = \mathrm{E}[X] + \mathrm{E}[Y]$. This is not always the case for variance. The following exercise demonstrates a sufficient condition for this kind of relationship to hold.

(a) In the notes the fact that for independent r.v.'s X and Y, $V[X+Y] = V[X] + V[Y]$ was used. Prove this result formally.

(b) Let X be the result of one fair six sided die. Let $Y = X$. (So Y and X are the result of the *same* die.) What is $V[X]$? $V[Y]$? $V[X+Y]$?

4. We say that a deck has k *face card adjacencies* if k of the face cards are followed by another of the same face card. (When a jack is followed by another jack, a queen is followed by another queen, or a king is followed by another king.) If all the kings are adjacent, all the queens are adjacent, and all the jacks are adjacent, this gives nine total adjacancies.

What is the expected number of face card adjacencies in a standard 52-card deck, shuffled into a permutation uniformly at random?