Section 11

1. Thinking rather than tabulating

(a) Is it always true that if X and Y are independently distributed (in the sense that, for any x and y, the event that $X = x$ and the event that $Y = y$ are independent), then $E[XY] = E[X]E[Y]$? Explain why or give an explicit counterexample joint distribution.

(b) Is it always true that if $E[XY] = E[X]E[Y]$, then X and Y are independently distributed? Explain why or give an explicit counterexample joint distribution.

(c) Is it always true that if X and Y are independently distributed, Y and Z are independently distributed, and X and Z are independently distributed, then $E[XYZ] = E[X]E[Y]E[Z]$? Explain why or give an explicit counterexample joint distribution.

2. Public Service Announcement

John is trying to get home, but can't remember the way. This is because he is drunk... so drunk that he doesn't realize he's already at home!

John makes two moves, starting from his house. Each time John moves, he independently chooses between moving one mile north, south, east, or west, each equally likely.

Let the random variable A be the number of miles north of home, and let the random variable B be the number of miles east of home, where John ends up after his two moves. (Consider south and west as negative, ignore the curvature of the Earth, and always drink in moderation)

(a) What is the joint distribution of A and B? Are A and B independently distributed?

(b) What is the marginal distribution of A given $B = 0$?

(c) What is the variance of A given $B > 0$?

(d) What is the expected value of $A + B$ given $B > 0$?

3. Tabulating

Consider the following table from lecture, in which each cell indicates how often player X plays the move indicated on the left and player Y plays the move indicated at the top:

X \ Y	Rock	Paper	Scissors
Rock	0.12	0.12	0.16
Paper	0.09	0.09	0.12
Scissors	0.09	0.09	0.12

Suppose, after being shown this chart, you are told that player Y failed to beat player X in their most recent game. Write out a joint probability distribution for player X and player Y's moves in their most recent game, by conditionalizing the chart on this new information.

4. Die, Die, Die!

A die is rolled three times. Let S be the sum of the first two rolls and let R be the sum of the last two rolls.

(a) What is the probability that $R = 4$ given $S = 2$?

(b) What is the probability that $R = 4$ given $S < 7$?

(c) What is the marginal distribution of S given $R = 4$?

(d) Given that $S + R = 6$, what is the probability $R = 2$?