Section 13

1. **[Countability]**
   Consider degree-one polynomials, i.e. polynomials of the form $P(x) = ax + b$. Determine if the set of all degree-one polynomials is countable under the following conditions:

   (a) a and b must be integers.

   (b) a and b must be rational numbers.

   (c) a and b are real numbers.

2. **[Cartesian products]**

   (a) Consider the set $\mathbb{N}^2 = \{(x, y) \mid x, y \in \mathbb{N}\}$ of pairs of natural numbers. Prove that $\mathbb{N}^2$ is countable.

   (b) Let n be a fixed positive integer. Consider the set $\mathbb{N}^n = \{(x_1, \ldots, x_n) \mid x_1, \ldots, x_n \in \mathbb{N}\}$ of n-tuples of natural numbers. Prove that $\mathbb{N}^n$ is countable.

3. **[Countable union of countable sets]**
   Suppose $X_1, \ldots, X_n, \ldots$ is an infinite sequence of countable sets. (In other words, for each positive integer $i \in \mathbb{Z}^+$, there is a countable set X_i.) Prove that their union $\cup_{i \in \mathbb{Z}^+} X_i$ is countable.

   [Remark: You may assume the X_i's are disjoint. The general case is not much harder.]

4. **[Wile E. Coyote]**
   Wile E. Coyote just bought some new Acme bombs and wants to drop one on the roadrunner. Wile E. Coyote hypothesizes that the roadrunner is at some location p, moving at some velocity v, and accelerating with constant acceleration a, where $p, v, a \in \mathbb{N}$. If this hypothesis is true, devise a strategy by which Wile E. Coyote is guaranteed to drop an Acme bomb on the roadrunner within a finite amount of time.